
The Importance of the TeV Scale

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Lecture 3

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The Standard Model Works

- Any discussion of the Standard Model has to start with its success
 - This is unlikely to be an accident!
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Issues with the Standard Model

- Unitarity
 - Landau pole
 - Triviality
 - Dependence of Higgs mass on high scale physics
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Unitarity

- Consider $2 \rightarrow 2$ elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) a_l$$

- a_l are the spin / partial waves
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Unitarity

- $P_l(\cos\theta)$ are Legendre polynomials:

$$\int_{-1}^1 dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$$

$$\sigma = \frac{8\pi}{s} \sum_{l=0}^{\infty} (2l+1) \sum_{l'=0}^{\infty} (2l'+1) a_l a_{l'}^* \int_{-1}^1 d \cos \theta P_l(\cos \theta) P_{l'}(\cos \theta)$$

$$= \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2$$

Sum of positive definite terms

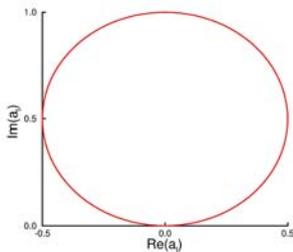
More on Unitarity

- Optical theorem $\sigma = \frac{1}{s} \text{Im}[A(\theta = 0)] = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l + 1) |a_l|^2$

$$\text{Im}(a_l) = |a_l|^2$$

Optical theorem derived
assuming only conservation
of probability

- Unitarity requirement:



$$|\text{Re}(a_l)| \leq \frac{1}{2}$$

More on Unitarity

- Idea: Use unitarity to limit parameters of theory

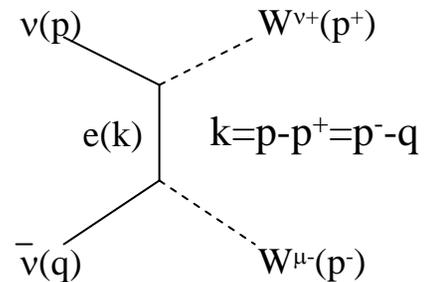
Cross sections which grow with energy always violate unitarity at some energy scale

Consider W^+W^- pair production

Example: $\nu\bar{\nu} \rightarrow W^+W^-$

➤ t-channel amplitude:

$$A_t(\nu\bar{\nu} \rightarrow W^+W^-) = -i \frac{g^2}{8} \bar{\nu}(q) \gamma^\mu (1 - \gamma_5) \frac{k}{k^2} \gamma^\nu (1 - \gamma_5) u(p) \varepsilon_\mu(p^-) \varepsilon_\nu(p^+)$$



➤ In center-of-mass frame:

$$p = \frac{\sqrt{s}}{2} (1, 0, 0, 1)$$

$$q = \frac{\sqrt{s}}{2} (1, 0, 0, -1)$$

$$p^+ = \frac{\sqrt{s}}{2} (1, 0, \beta_W \sin \theta, \beta_W \cos \theta)$$

$$p^- = \frac{\sqrt{s}}{2} (1, 0, -\beta_W \sin \theta, -\beta_W \cos \theta)$$

$$s = (p + q)^2$$

$$t = k^2 = (p - p')^2$$

$$\beta_W = \sqrt{1 - 4M_W^2 / s}$$

W^+W^- pair production, 2

- Interesting physics is in the longitudinal W sector:

$$A_t(\nu\bar{\nu} \rightarrow W_L^+W_L^-) = i\frac{g^2}{4M_W^2}\bar{\nu}(q)k(1+\gamma_5)u(p)$$

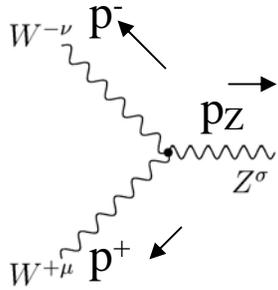
$$\varepsilon^+ \rightarrow \frac{p^+}{M_W} + O\left(\frac{M_W^2}{s}\right)$$

- Use Dirac Equation: $\not{p}u(p)=0$

$$\longrightarrow \left|A_t(\nu\bar{\nu} \rightarrow W_L^+W_L^-)\right|^2 = 2G_F^2 s^2 \sin^2 \theta + O\left(\frac{M_W^2}{s}\right)$$

Grows with energy

Feynman Rules for Gauge Boson Vertices

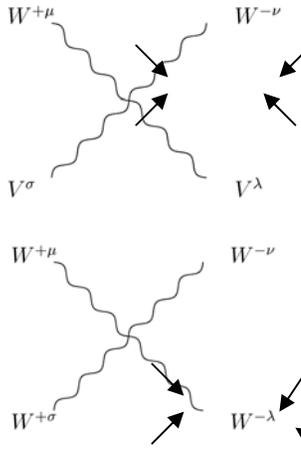


$$ig_{WWV} V^{\mu\nu\sigma}(p^+, p^-, p_Z)$$

$$g_{WW\gamma} = e$$

$$g_{WWZ} = e \cot \theta_W$$

$$V^{\mu\nu\sigma}(p^+, p^-, p_Z) = (p^+ - p^-)^\sigma g^{\mu\nu} + (p^- - p_Z)^\mu g^{\sigma\nu} + (p_Z - p^+)^\nu g^{\mu\sigma}$$



$$-ig_{WWWV} (2g^{\sigma\lambda} g^{\mu\nu} - g^{\sigma\mu} g^{\lambda\nu} - g^{\lambda\mu} g^{\sigma\nu})$$

$$ig^2 (2g^{\sigma\mu} g^{\lambda\nu} - g^{\mu\nu} g^{\lambda\sigma} - g^{\lambda\mu} g^{\sigma\nu})$$

$$g_{WW\gamma\gamma} = e^2$$

$$g_{WW\gamma Z} = e^2 \cot \theta_W$$

$$g_{WWZZ} = e^2 \cot^2 \theta_W$$

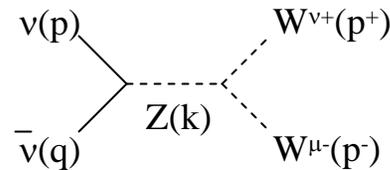
W⁺W⁻ pair production, 3

- SM has additional contribution from s-channel Z exchange

$$A_s(\nu\bar{\nu} \rightarrow W^+W^-) = -i \frac{g^2}{4(s-M_Z^2)} \bar{\nu}(q) \gamma_\mu (1-\gamma_5) u(p) \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M_Z^2} \right) \left[g_{\lambda\rho} (p^- - p^+)_\nu + g_{\lambda\nu} (p^+ + k)_\rho - g_{\rho\nu} (p^- + k)_\lambda \right] \varepsilon^\lambda(p^+) \varepsilon^\rho(p^-)$$

- For longitudinal W's

$$A_s(\nu\bar{\nu} \rightarrow W_L^+ W_L^-) = i \frac{g^2}{4M_W^2} \bar{\nu}(q) (p^+ - p^-) (1-\gamma_5) u(p)$$



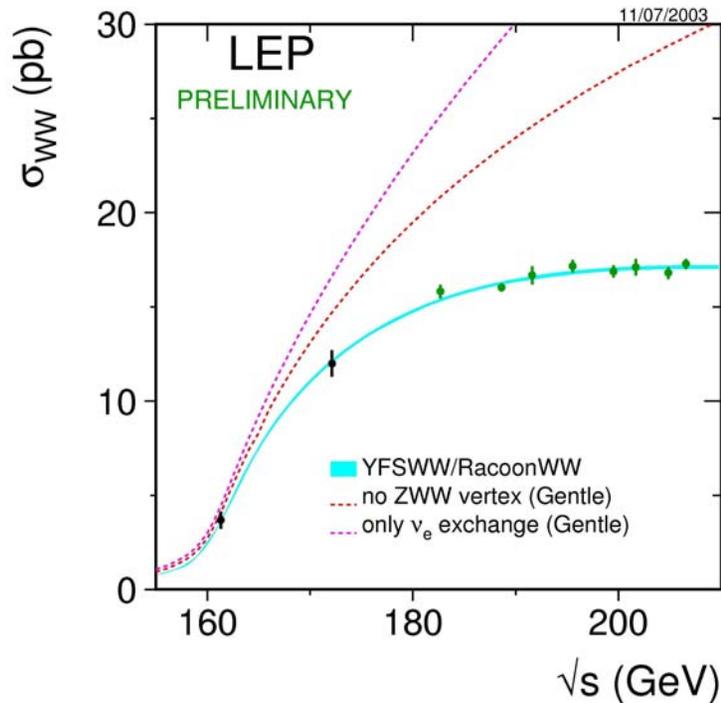
$$A_s(\nu\bar{\nu} \rightarrow W_L^+ W_L^-) = -i \frac{g^2}{4M_W^2} \bar{\nu}(q) k (1+\gamma_5) u(p)$$

Contributions which grow with energy cancel between t- and s-channel diagrams



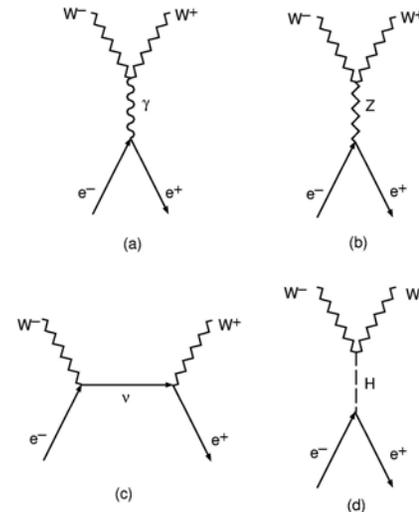
Depends on special form of 3-gauge boson couplings

No deviations from SM at LEP2



LEP EWWG, hep-ex/0312023

No evidence for Non-SM
3 gauge boson vertices



Contribution which grows
like $m_e^2 s$ cancels between
Higgs diagram and others

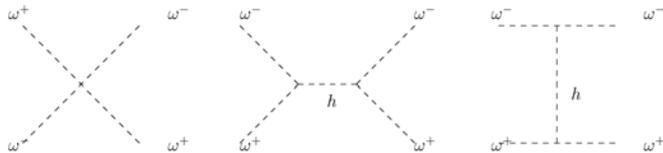
Example: $W^+W^- \rightarrow W^+W^-$

- Recall scalar potential (Include Goldstone Bosons)

$$V = \frac{M_h^2}{2} h^2 + \frac{M_h^2}{2v} h(h^2 + z^2 + 2\omega^+\omega^-) + \frac{M_h^2}{8v^2} (h^2 + z^2 + 2\omega^+\omega^-)^2$$

- Consider Goldstone boson scattering:

$$\omega^+\omega^- \rightarrow \omega^+\omega$$



$$iA(\omega^+\omega^- \rightarrow \omega^+\omega^-) = -2i \frac{M_h^2}{v^2} + \left(-i \frac{M_h^2}{v} \right)^2 \frac{i}{t - M_h^2} + \left(-i \frac{M_h^2}{v} \right)^2 \frac{i}{s - M_h^2}$$

$$\omega^+ \omega^- \rightarrow \omega^+ \omega^-$$

■ Two interesting limits:

□ $s, t \gg M_h^2$

$$A(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) \rightarrow -2 \frac{M_h^2}{v^2}$$

$$a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}$$

□ $s, t \ll M_h^2$

$$A(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) \rightarrow -\frac{u}{v^2}$$

$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

Use Unitarity to Bound Higgs

$$|\operatorname{Re}(a_l)| \leq \frac{1}{2}$$

- High energy limit:

$$a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}$$

$$M_h < 800 \text{ GeV}$$

- Heavy Higgs limit

$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

$$E_c \sim 1.7 \text{ TeV}$$

→ New physics at the TeV scale

Can get more stringent bound from coupled channel analysis

Electroweak Equivalence Theorem

$$A(V_L^1 \dots V_L^N \rightarrow V_L^1 \dots V_L^{N'}) = (i)^N (-i)^{N'} A(\omega_1 \dots \omega_N \rightarrow \omega_1 \dots \omega_{N'}) \\ + \mathcal{O}\left(\frac{M_W^2}{E^2}\right)$$

This is a statement about scattering amplitudes, NOT individual Feynman diagrams

Landau Pole

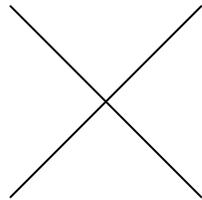
- M_h is a free parameter in the Standard Model
- Can we derive limits on the basis of consistency?
- Consider a scalar potential:

$$V = \frac{M_h^2}{2} h^2 + \frac{\lambda}{4} h^4$$

- This is potential at electroweak scale
 - Parameters evolve with energy in a calculable way
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Consider $hh \rightarrow hh$

- Real scattering, $s+t+u=4M_h^2$
- Consider momentum space-like and off-shell:
 $s=t=u=Q^2<0$
- Tree level: $iA_0=-6i\lambda$

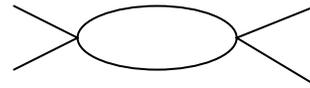


hh→hh, #2

- One loop:

$$\begin{aligned} iA_s &= (-6i\lambda)^2 \frac{1}{2} \int \frac{d^n k}{(2\pi)^2} \frac{i}{k^2 - M_h^2} \frac{i}{(k+p+q)^2 - M_h^2} \\ &= \frac{9\lambda^2}{8\pi^2} (4\pi\mu^2) \Gamma(\varepsilon) (M_h^2 - Q^2 x(1-x))^{-\varepsilon} \end{aligned}$$

- $A = A_0 + A_s + A_t + A_u$



$$A = -6\lambda \left(1 + \frac{9\lambda}{16\pi^2} (4\pi\mu^2) \Gamma(\varepsilon) (M_h^2 - Q^2 x(1-x))^{-\varepsilon} + \dots \right)$$

hh→hh, #3

- Sum the geometric series to define running coupling

$$A = -6\lambda \left(1 + \frac{9\lambda}{16\pi^2} \log \frac{Q^2}{M_h^2} \right) + \dots$$

$$A = \frac{6\lambda}{1 - \frac{9\lambda}{8\pi^2} \log \left(\frac{Q}{M_h} \right)} \equiv 6\lambda(Q)$$

- $\lambda(Q)$ blows up as $Q \rightarrow \infty$ (called Landau pole)

hh→hh, #4

- This is independent of starting point
- BUT.... Without $\lambda\phi^4$ interactions, theory is non-interacting
- Require quartic coupling be finite

$$\frac{1}{\lambda(Q)} > 0$$

hh→hh, #5

- Use $\lambda = M_h^2 / (2v^2)$ and approximate $\log(Q/M_h) \rightarrow \log(Q/v)$
- Requirement for $1/\lambda(Q) > 0$ gives upper limit on M_h

$$M_h^2 < \frac{32\pi^2 v^2}{9 \log\left(\frac{Q^2}{v^2}\right)}$$

- Assume theory is valid to 10^{16} GeV
 - Gives upper limit on $M_h < 180$ GeV
- Can add fermions, gauge bosons, etc.

High Energy Behavior of λ

- Renormalization group scaling $\frac{1}{\lambda(Q)} = \frac{1}{\lambda(\mu)} + (\dots) \log\left(\frac{Q}{\mu}\right)$

$$16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12g_t^4 + (\text{gauge})$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right)$$

$$g_t = \frac{M_t}{v}$$

- *Large λ (Heavy Higgs)*: self coupling causes λ to grow with scale
- *Small λ (Light Higgs)*: coupling to top quark causes λ to become negative

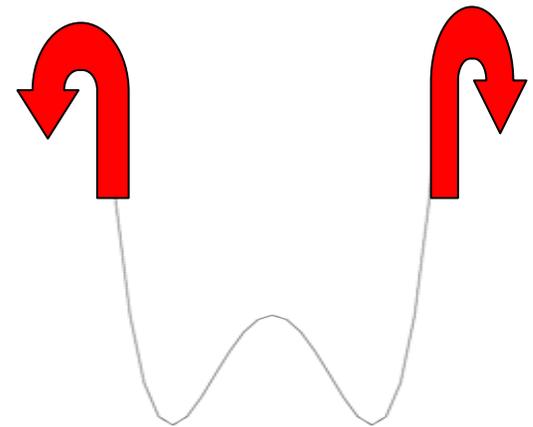
Does Spontaneous Symmetry Breaking Happen?

- SM requires spontaneous symmetry
- This requires $V(v) < V(0)$
- For small λ

$$16\pi^2 \frac{d\lambda}{dt} \approx -16g_t^4$$

- Solve

$$\lambda(\Lambda) \approx \lambda(v) - \frac{3g_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)$$



Does Spontaneous Symmetry Breaking Happen? (#2)

- $\lambda(\Lambda) > 0$ gives lower bound on M_h

$$M_h^2 > \frac{3v^2}{2\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)$$

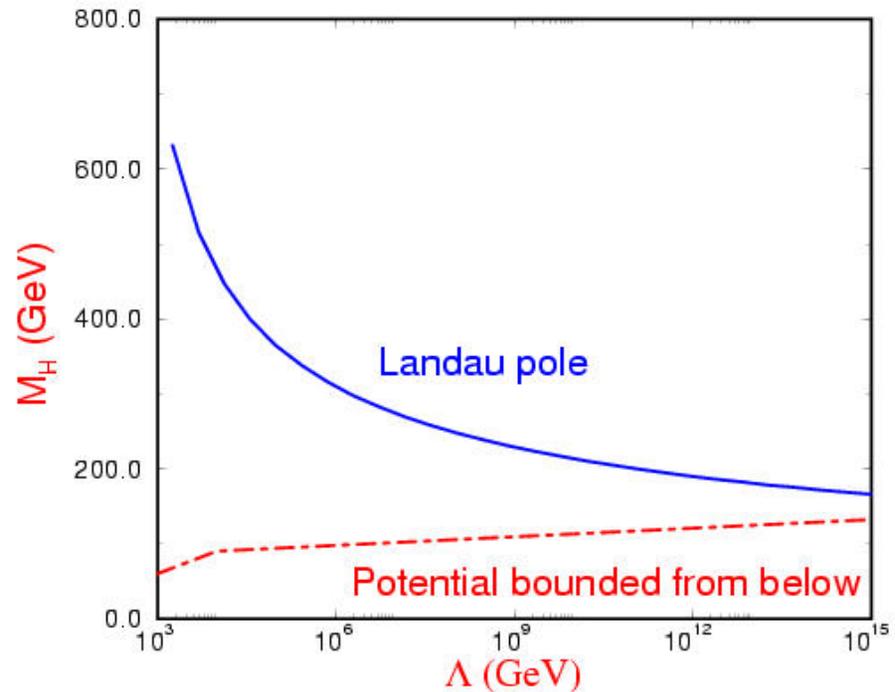
- If Standard Model valid to 10^{16} GeV

$$M_h > 130 \text{ GeV}$$

- For any given scale, Λ , there is a theoretically consistent range for M_h

Bounds on SM Higgs Boson

- If SM valid up to Planck scale, only a small range of allowed Higgs Masses



More Problems

- We often say that the SM cannot be the entire story because of the quadratic divergences of the Higgs Boson mass



Masses at one-loop

- First consider a fermion coupled to a massive complex Higgs scalar

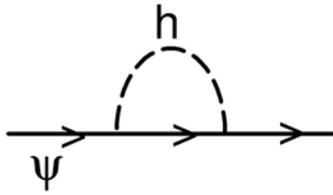
$$L = \bar{\Psi}(i\partial)\Psi + |\partial_\mu\phi|^2 - m_s|\phi|^2 - (\lambda_F \bar{\Psi}_L \Psi_R \phi + h.c.)$$

- Assume symmetry breaking as in SM:

$$\phi = \frac{(h+v)}{\sqrt{2}} \qquad m_F = \frac{\lambda_F v}{\sqrt{2}}$$

Masses at one-loop, #2

- Calculate mass renormalization for Ψ



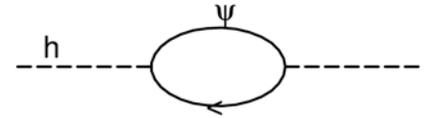
$$\delta m_F = -\frac{3\lambda_F^2 m_F}{32\pi^2} \log\left(\frac{\Lambda^2}{m_F^2}\right) + \dots$$

Symmetry and the fermion mass

- $\delta m_F \approx m_F$
 - $m_F=0$, then quantum corrections vanish
 - When $m_F=0$, Lagrangian is invariant under
 - $\Psi_L \rightarrow e^{i\theta_L} \Psi_L$
 - $\Psi_R \rightarrow e^{i\theta_R} \Psi_R$
 - $m_F \rightarrow 0$ increases the symmetry of the theory
 - Yukawa coupling (proportional to mass) breaks symmetry and so corrections $\approx m_F$

Scalars are very different

$$\delta M_h^2 = \Sigma_S(m_s^2) = -\frac{\lambda_F^2 \Lambda^2}{8\pi^2} + (m_s^2 - m_F^2) \log\left(\frac{\Lambda}{m_F}\right) \\ + (2m_F^2 - \frac{m_s^2}{2}) \left(1 + I_1\left(\frac{m_s^2}{m_F^2}\right)\right) + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$



$$I_1(a) = \int_0^1 dx \log(1 - ax(1-x))$$

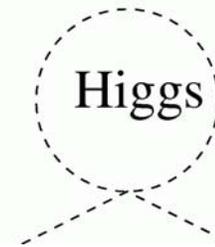
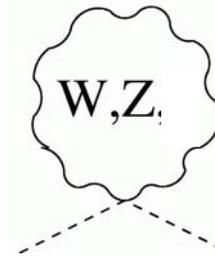
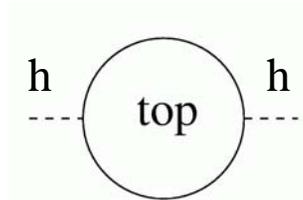
- M_h diverges quadratically!
- This implies quadratic sensitivity to high mass scales

Scalars (#2)

- M_h diverges quadratically!
 - Requires large cancellations (hierarchy problem)
 - Can do this in Quantum Field Theory
 - h does not obey decoupling theorem
 - Says that effects of heavy particles decouple as $M \rightarrow \infty$
 - $M_h \rightarrow 0$ doesn't increase symmetry of theory
 - **Nothing protects Higgs mass from large corrections**
-

Light Scalars are Unnatural

- Higgs mass grows with scale of new physics, Λ
- No additional symmetry for $M_h=0$, no protection from large corrections



$$\begin{aligned}\delta M_h^2 &= \frac{G_F}{4\sqrt{2}\pi^2} \Lambda^2 (6M_W^2 + 3M_Z^2 + M_h^2 - 12M_t^2) \\ &= -\left(\frac{\Lambda}{0.7 \text{ TeV}} 200 \text{ GeV} \right)^2\end{aligned}$$

$M_h \leq 200 \text{ GeV}$ requires large cancellations

What's the problem?

- Compute M_h in dimensional regularization and absorb infinities into definition of M_h

$$M_h^2 = M_{h0}^2 + \frac{1}{\epsilon}(\dots)$$

- Perfectly valid approach
 - Except we know there is a high scale
-

Try to cancel quadratic divergences by adding new particles

- SUSY models add scalars with same quantum numbers as fermions, but different spin
 - Little Higgs models cancel quadratic divergences with new particles with same spin
-

We expect something at the TeV scale

- If it's a SM Higgs then we have to think hard about what the quadratic divergences are telling us
 - SM Higgs mass is highly restricted by requirement of theoretical consistency
-